THE KING'S SCHOOL, CANTERBURY

SCHOLARSHIP ENTRANCE EXAMINATION

March 2010

MATHEMATICS Paper II

Time: 1 hour (plus reading time)

Use the reading time wisely; gain an overview of the paper and start to think of how you will answer the questions.

Do as many questions as you can (clearly numbered) on the lined paper provided.

The questions are not of equal length or mark allocation. Make sure you avoid spending too much time on any one question; don't get bogged down! Move on quickly if you get stuck. The paper is quite long; you are not necessarily expected to finish everything.

Some of the later questions are more difficult, but not necessarily longer. Some questions are designed to test your ability to work with unfamiliar ideas, or familiar ones with a twist. Don't give up!

You are expected to use a calculator where appropriate, but also you must show ***full and clear working****, diagrams and arguments wherever you can. Marks will be awarded for method as well as answers. In fact, merely writing down an answer might score few marks.*

Complete questions are preferable to fragments. You can sometimes, however, manage to complete later parts of questions, even if you have failed to answer the earlier sections.

This paper has ten questions and six pages.

1 In 2009 the Edinburgh Festival Fringe – the world's largest arts festival - featured approximately 2,200 different shows in 256 different venues.

In total there were **34,265** performances.

The average length of a show was about **fifty minutes**.

Suppose we were somehow able to film every performance of every show.

Estimate, showing your working, how long this film would last in total.

[You must give your answer in appropriate units.]

2 [In this question (involving organised counting), try a more sophisticated approach than merely writing out all the possibilities.]

Captain Bellybuster goes to Ivan's Corner Café opposite King's for a fried breakfast.

Menu 1
Bacon
Egg
Sausage
Black Pudding

(a) He decides to choose two items from Menu 1 above. Explain why there are six possible different breakfasts he can choose.

(b) Next time Captain Bellybuster visits he is a lot hungrier. He orders from Menu 2 below.

Menu 2 **Two of**	**Two of**	**One of**	**One of**
Bacon	**Mushrooms**	**Tea**	**Toast**
Egg	**Beans**	**Coffee**	**Bread**
Sausage	**Tomato**		
Black Pudding	**Fried Slice**		
	Hash Browns		

Showing your working, work out how many essentially different breakfasts he can choose from Menu 2.

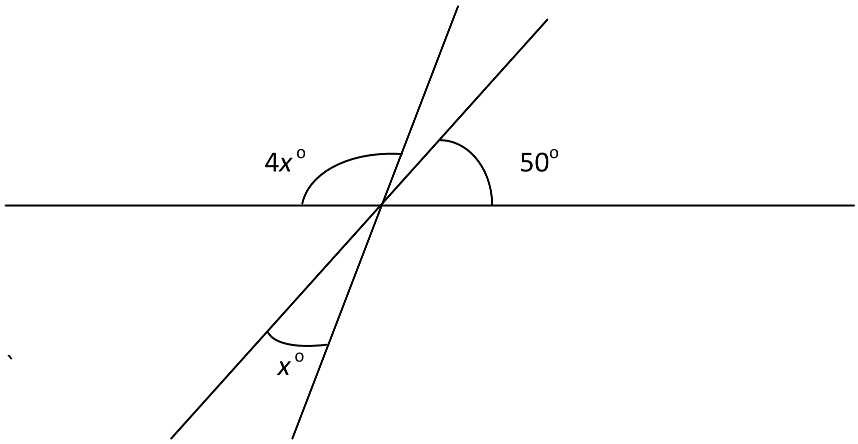
3 The celebrated writer, Jonathan Swift (30 November 1667 – 19 October 1745), wrote in a letter to Alexander Pope on 7 Feb. 1736:

"What vexes me most is, that my female friends, who could bear me very well a dozen years ago, have now forsaken me, although I am not so old in proportion to them as I formerly was: which I can prove by arithmetic, for then I was double their age, which now I am not."

Try and see if you can "prove by arithmetic" i.e. if we assume he means exactly twelve years ago, what is the precise ratio (**in lowest terms**) of his and the women's ages **now**?

[Hint: use the dates given in the question]

4 Work out the value of x from the diagram below (three straight lines intersecting).



5 Tommy drives his Toyota on a long journey.

- He drives 10 miles at speed v mph
- Then the brakes stop working and he drives the next 40 miles at speed $2v$ mph.
- Then the accelerator pedal sticks down and he drives the next 90 miles at a speed of $3v$ mph.

What is Tommy's average speed (in terms of v, in mph) for the whole journey?

6 The philosopher William James once said:

"Whenever two men meet there are really six people present. There is each man as he sees himself, each man as the other sees him, and each man as he really is."

(a) Following James' reasoning, explain why, if three men meet, there are really twelve people present.

(b) How many are really present if four men meet?

(c) Find a formula, in terms of n (number of men who meet), for P (number really present).

[Hint: you might try drawing a diagram or similar]

(d) How many men meet if there were 156 people really present?

PTO

7 In yesterday's paper we met Farey sequences. Here they are again:

F_1 $$\frac{0}{1} \quad \frac{1}{1}$$

F_2 $$\frac{0}{1} \quad \frac{1}{2} \quad \frac{1}{1}$$

F_3 $$\frac{0}{1} \quad \frac{1}{3} \quad \frac{1}{2} \quad \frac{2}{3} \quad \frac{1}{1}$$

F_4 $$\frac{0}{1} \quad \frac{1}{4} \quad \frac{1}{3} \quad \frac{1}{2} \quad \frac{2}{3} \quad \frac{3}{4} \quad \frac{1}{1}$$

F_5 $$\frac{0}{1} \quad \frac{1}{5} \quad \frac{1}{4} \quad \frac{1}{3} \quad \frac{2}{5} \quad \frac{1}{2} \quad \frac{3}{5} \quad \frac{2}{3} \quad \frac{3}{4} \quad \frac{4}{5} \quad \frac{1}{1}$$

Reminder: the n^{th} Farey Sequence, F_n, has all the fractions between 0 and 1 (inclusive), **in order of size**, and **in lowest terms**, with denominators up to and including n.

(a) Explain carefully why the next Farey sequence will always be longer than the last one. Can you say certain types of fractions which will **always** appear in the next row?

(b) Consider any three consecutive fractions from any Farey sequence:

$$\cdots \quad \frac{a}{b} \quad \frac{?}{?} \quad \frac{c}{d} \quad \cdots$$

Can you write down, algebraically, what you think the middle fraction should always be?

Here is the tenth Farey Sequence.

F_{10} $$\frac{0}{1} \; \frac{1}{10} \; \frac{1}{9} \; \frac{1}{8} \; \frac{1}{7} \; \frac{1}{6} \; \frac{1}{5} \; \frac{2}{9} \; \frac{1}{4} \; \frac{2}{7} \; \frac{3}{10} \; \frac{1}{3} \; \frac{3}{8} \; \frac{2}{5} \; \frac{3}{7} \; \frac{4}{9} \; \frac{1}{2} \; \frac{5}{9} \; \frac{4}{7} \; \frac{3}{5} \; \frac{5}{8} \; \frac{2}{3} \; \frac{7}{10} \; \frac{5}{7} \; \frac{3}{4} \; \frac{7}{9} \; \frac{4}{5} \; \frac{5}{6} \; \frac{6}{7} \; \frac{7}{8} \; \frac{8}{9} \; \frac{9}{10} \; \frac{1}{1}$$

(c) Which new fractions get added in here when we write down F_{11} ? [Don't write out the whole sequence]

(d) Which new fractions get added into F_{11} when we write down F_{12} ? [Don't write out the whole sequence]

(e) Explain carefully why we don't add so many new ones in (d) as we do in (c) .

8 You win £1 million in a game show. The host reveals two bowls with five balls in each (one group blue, one green) and gives you a choice:

Mix the balls up in any way you like between the two bowls.

Then you are blindfolded and the bowls are moved around so you don't know which is which.

You choose a bowl at random and pick out a ball.

If it is blue you win £10 million; if green, you lose everything.

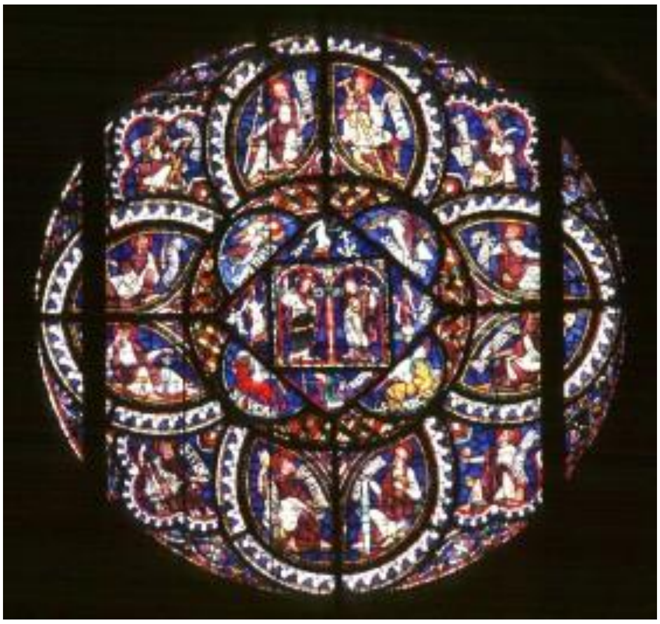
So, what do you do when you mix them up so you can maximise your chance of winning? Explain your answer carefully.

9 Albert walks to visit his friend Bob and then returns home by exactly the same route. The total journey (there-and-back) takes six hours.

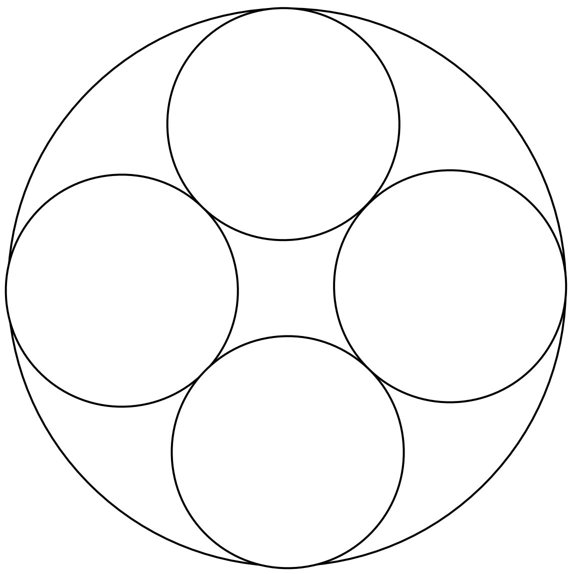
- He walks uphill at 2 km/h
- He walks downhill at 6 km/h
- He walks at 3 km/h on the flat.

How far does Albert walk in total? [You should show your working carefully and give your answer in the most appropriate units.]

10 Here is the South Oculus window in Canterbury Cathedral:



Let's look more closely at the ferramenta (the metal frames). Here is a simplified version:



(a) If the radius of the smaller, touching, circles is exactly 1 metre. Show that the radius of the larger circle is

$$1 + \sqrt{2} \quad \text{metres}$$

PTO

(b) Show that the area of the larger circle is $\pi(3 + 2\sqrt{2})$

From (a) we know the diameter of the larger circle is $2 + 2\sqrt{2}$.

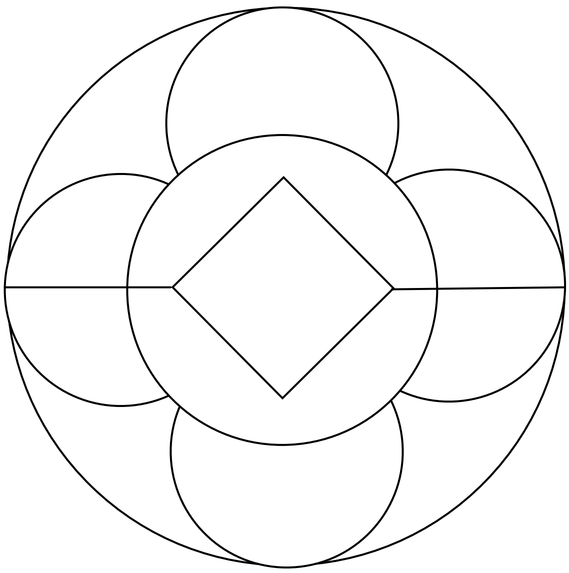
This means that the square in the centre (see below), has **diagonal** of length one-third the diameter, i.e.

$$\frac{2 + 2\sqrt{2}}{3}$$

(c) Explain carefully why a square with diagonal *x* has area $\frac{1}{2}x^2$

(d) Find and simplify an expression for the area of the square at the centre of the window.

(e) Find the percentage (to some accuracy) of the larger circle occupied by the square in the middle.



END OF EXAMINATION